Prova de calculo
#***Thiago gentil ramires***

#assumindo K=1

#***P****rimeira Questão*

$$f := x \to \left(\frac{1}{\Gamma\left(\frac{1}{2}\right)}\right)\cdot\left(\frac{1}{2}\right)^{\frac{1}{2}}\cdot x^{\frac{1}{2}-1}\cdot \exp\left(-\frac{x}{2}\right);$$

#assumindo a constante = 1 temos

$$c := \left(\frac{1}{\Gamma\left(\frac{1}{2}\right)}\right)\cdot\left(\frac{1}{2}\right)^{\frac{1}{2}};$$

$$d := x \to x^{\frac{1}{2}-1}\cdot \exp\left(-\frac{1}{2}\right);$$

$$x \to \frac{1}{2}\,\frac{\sqrt{2}\,e^{-\frac{1}{2}x}}{\Gamma\left(\frac{1}{2}\right)\sqrt{x}}$$

$$\frac{1}{2}\,\frac{\sqrt{2}}{\sqrt{\pi}}$$

$$x \to \frac{e^{-\frac{1}{2}}}{\sqrt{x}} \qquad \textbf{(1)}$$

#descontinuidade;
discont$(f(x), x)$

$$\{0\} \qquad \textbf{(2)}$$

#Interceptos:
$f(0)$

#Raizes
solve$(f(x), x);$
#Não possui candidato à assíntota vertical

#Assíntotas Horizontais:
Limit$(f(x), x = infinity) =$ *limit*$(f(x), x = infinity);$

$$\lim_{x\to\infty} \frac{1}{2} \frac{\sqrt{2}\, e^{-\frac{1}{2}x}}{\sqrt{\pi}\sqrt{x}} = 0$$

$$\lim_{x\to\infty} \frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k-1} e^{-\frac{1}{2}x}}{\Gamma\left(\frac{1}{2}k\right)} = 0 \qquad \textbf{(4)}$$

$Limit(f(x), x=0, right) = limit(f(x), x=0, right);$

$$\lim_{x\to 0+} \frac{1}{2} \frac{\sqrt{2}\, e^{-\frac{1}{2}x}}{\sqrt{\pi}\sqrt{x}} = \infty \qquad \textbf{(5)}$$

```
> #Logo, a funçao f não possui assíntotas horizontais
> #Crescimento e decrescimento/ Máximos e mínimos:
> #Encontrando a raiz da derivada de primeira ordem temos:
> d1:=x->diff(f(x),x):d1(x);
```

$$-\frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{2}x}}{\sqrt{\pi}\, x^{3/2}} - \frac{1}{4} \frac{\sqrt{2}\, e^{-\frac{1}{2}x}}{\sqrt{\pi}\sqrt{x}}$$

$solve(d1(x)=0);$

$$-1$$

$$\{k = 2 + x, x = x\}$$

$$\{k = 2 + x, x = x\} \qquad \textbf{(8)}$$

```
> #Intervalos de Decrescimento:
> solve(d1<0);
```

$$RealRange(-\infty, Open(0))$$

```
> #Intervalos de Crescimento
> solve(d1>0);
```

$$RealRange(Open(0), \infty)$$

```
> #Concavidade:
```

$> d2 := diff(f(x), x\$2) : d2;$

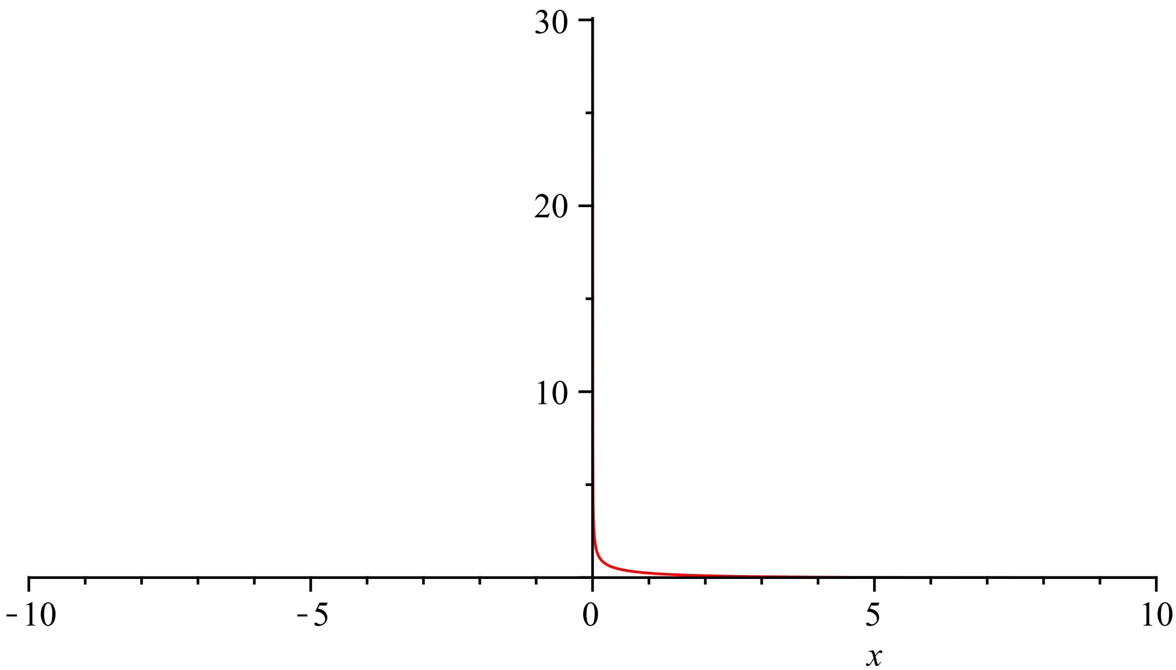
$$\frac{3}{8}\,\frac{\sqrt{2}\,e^{-\frac{1}{2}x}}{\sqrt{\pi}\,x^{5/2}}+\frac{1}{4}\,\frac{\sqrt{2}\,e^{-\frac{1}{2}x}}{\sqrt{\pi}\,x^{3/2}}+\frac{1}{8}\,\frac{\sqrt{2}\,e^{-\frac{1}{2}x}}{\sqrt{\pi}\,\sqrt{x}} \tag{9}$$

simplify(%);

$$\frac{1}{8}\,\frac{\sqrt{2}\,e^{-\frac{1}{2}x}\left(3+2x+x^2\right)}{\sqrt{\pi}\,x^{5/2}} \tag{10}$$

```
> #Funçao concavo pra cima:
> solve(d2>0,x);
Warning, solutions may have been lost
> #Funçao concavo pra baixo:
> solve(d2<0,x);
Warning, solutions may have been lost
```

plot(*f* (*x*));



#B)

$c\cdot Int(c\cdot d(x), x=0..infinity) = int(f(x)\cdot c, x=0..infinity,) :$

simplify(%);

$$\frac{2^{-\frac{1}{2}k}\left(\int_0^{\infty}\frac{2^{-\frac{1}{2}k}x^{\frac{1}{2}k-1}e^{-\frac{1}{2}x}}{\Gamma\left(\frac{1}{2}k\right)}\,dx\right)}{\Gamma\left(\frac{1}{2}k\right)}=\frac{2^{-\frac{1}{2}k}}{\Gamma\left(\frac{1}{2}k\right)} \tag{11}$$

#pelo calculo de gama temos que $2^{-\frac{1}{2}k}=\Gamma\left(\frac{1}{2}k\right)$: *Portanto a integral da função Qui − quadrado de zero a* $+00$
 # é igual a 1

#C)
$c \cdot Int(d(x)\cdot x, x=0..infinity) = c\cdot int(d(x)\cdot x, x=0..infinity,);$

$$\frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k}\left(\int_0^{\infty}x^{\frac{1}{2}k-1}e^{-\frac{1}{2}x}x\,dx\right)}{\Gamma\left(\frac{1}{2}k\right)}=\frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k}2^{\frac{1}{2}k+1}\Gamma\left(\frac{1}{2}k+1\right)}{\Gamma\left(\frac{1}{2}k\right)} \tag{12}$$

simplify(%)

$$\frac{2^{-\frac{1}{2}k}\left(\int_0^{\infty}x^{\frac{1}{2}k}e^{-\frac{1}{2}x}\,dx\right)}{\Gamma\left(\frac{1}{2}k\right)}=k \tag{13}$$

 # Temos que I1=k

#D)
$c\cdot Int(d(x)\cdot x^2, x=0..infinity) = c\cdot int(d(x)\cdot x^2, x=0..infinity,);$

$$\frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k}\left(\int_0^{\infty}x^{\frac{1}{2}k-1}e^{-\frac{1}{2}x}x^2\,dx\right)}{\Gamma\left(\frac{1}{2}k\right)}=\frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k}2^{2+\frac{1}{2}k}\Gamma\left(2+\frac{1}{2}k\right)}{\Gamma\left(\frac{1}{2}k\right)} \tag{14}$$

simplify(%)

$$\frac{2^{-\frac{1}{2}k}\left(\int_0^{\infty}x^{\frac{1}{2}k+1}e^{-\frac{1}{2}x}\,dx\right)}{\Gamma\left(\frac{1}{2}k\right)}=k\,(k+2) \tag{15}$$

$I2 := c\cdot int(d(x)\cdot x^2, x=0..infinity,) :$
$I1 := c\cdot int(d(x)\cdot x, x=0..infinity,) :$

$var = I2 - (I1)^2;$

$$var = \frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k} 2^{2+\frac{1}{2}k} \Gamma\left(2+\frac{1}{2}k\right)}{\Gamma\left(\frac{1}{2}k\right)} - \frac{\left(\left(\frac{1}{2}\right)^{\frac{1}{2}k}\right)^2 \left(2^{\frac{1}{2}k+1}\right)^2 \Gamma\left(\frac{1}{2}k+1\right)^2}{\Gamma\left(\frac{1}{2}k\right)^2} \quad \textbf{(16)}$$

$simplify(\%)$

$$var = 2k \quad \textbf{(17)}$$

#E)

$c \cdot Int\left(d(x) \cdot e^{t \cdot x}, x = 0 ..infinity\right) = c \cdot int\left(d(x) \cdot e^{t \cdot x}, x = 0 ..infinity, \right);$

$$\frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k} \left(\int_0^{\infty} x^{\frac{1}{2}k-1} \mathrm{e}^{-\frac{1}{2}x} e^{tx} \, \mathrm{d}x\right)}{\Gamma\left(\frac{1}{2}k\right)} = \frac{\left(\frac{1}{2}\right)^{\frac{1}{2}k} \left(\int_0^{\infty} x^{\frac{1}{2}k-1} \mathrm{e}^{-\frac{1}{2}x} e^{tx} \, \mathrm{d}x\right)}{\Gamma\left(\frac{1}{2}k\right)} \quad \textbf{(18)}$$

$simplify(\%);$

$$\frac{2^{-\frac{1}{2}k} \left(\int_0^{\infty} x^{\frac{1}{2}k-1} \mathrm{e}^{-\frac{1}{2}x} e^{tx} \, \mathrm{d}x\right)}{\Gamma\left(\frac{1}{2}k\right)} = \frac{2^{-\frac{1}{2}k} \left(\int_0^{\infty} x^{\frac{1}{2}k-1} \mathrm{e}^{-\frac{1}{2}x} e^{tx} \, \mathrm{d}x\right)}{\Gamma\left(\frac{1}{2}k\right)} \quad \textbf{(19)}$$

$with(Student[Calculus1]) :$

#Segunda Questão

#A)

```
restart;
f := (x1, x2) -> 1;
```

$$(x1, x2) \to 1 \quad (20)$$

```
r1 := x -> x;
```

$$x \to x \quad (21)$$

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r2 := x -> -x;
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$$x \to -x \quad (22)$$

```
r3 := x -> x - 2;
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$$x \to x - 2 \quad (23)$$

```
r4 := x -> -x + 2;
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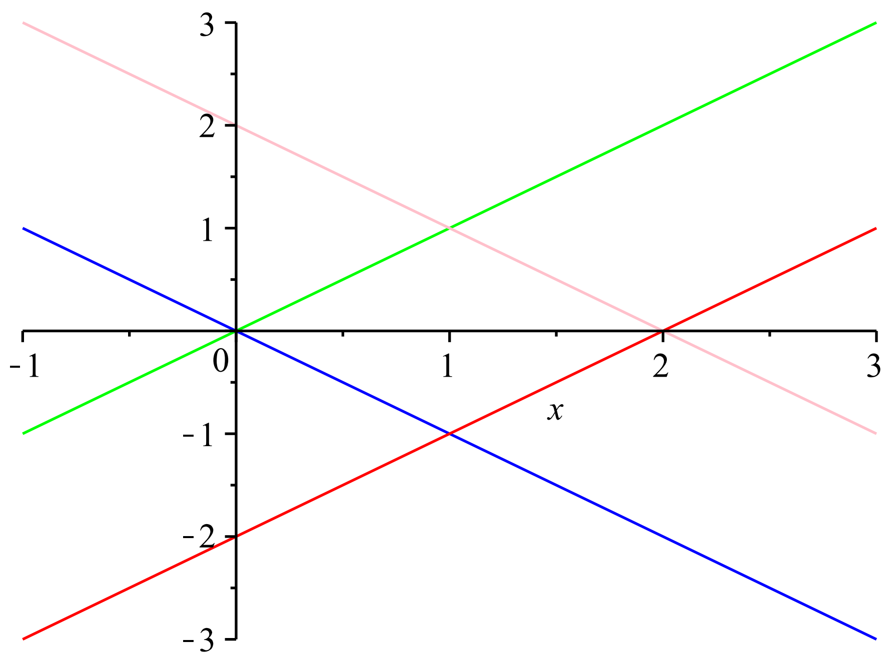
$$x \to -x + 2 \quad (24)$$

```
g1 := plot(r1(x), x=-1..3, color=green) :
g2 := plot(r2(x), x=-1..3, color=blue) :
g3 := plot(r3(x), x=-1..3, color=red) :
g4 := plot(r4(x), x=-1..3, color=pink) :
with(plots) :
display(g1, g2, g3, g4);
```



#B)

```
Int(1, x2=0..2) = int(1, x2=0..2);
```

$$\int_0^2 1 \, dx2 = 2 \quad (25)$$

#C)

$Int(1, x2 = -1 .. -1) = int(1, x2 = 0 .. 2);$

$$\int_{-1}^{-1} 1 \, \mathrm{d}x2 = 2 \qquad \textbf{(26)}$$