Radiometric analysis of the light coupled by optimally cut plastic optical fiber amplitude modulating reflectance displacement sensors

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A radiometric analysis of the light coupled by optical fiber amplitude modulating extrinsic-type reflectance displacement sensors is presented. Uncut fiber sensors show the largest range but a smaller responsivity. Single cut fiber sensors exhibit an improvement in responsivity at the expense of range. A further increase in responsivity as well as a reduction in the operational range is obtained when the double cut sensor configuration is implemented. The double cut configuration is particularly suitable in applications where feedback action is applied to the moving reflector surface.

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I. INTRODUCTION

Extrinsic-type displacement transducers that employ optical fibers to measure linear or angular displacement, velocity, acceleration, and pressure, as well as force, have been already discussed extensively in the literature.1–6 A common feature in all emitter–detector configurations which utilize a pair of fibers adjacent to each other is the response of the transducer proportional to the flux entering the receiving fiber. In close proximity to the reflector, a curve of intensity against fiber–reflector separation shows an approximately linear region whereas further away from the reflector, a second region, which follows an inverse square-law relation to the transmitting point, and ψ is the incidence angle of the ray on the receiving surface which is, in fact, equal to the exit angle θ from the transmitting surface as the two surfaces are parallel. A similar observation can be made for the azimuth of the ray from the transmitter surface so that ϕ = ω.

Calculations can then be performed after first assuming a point source P on the emitting fiber surface with coordinates P = (x_P, y_P, 0). A point Q at position (x_a, y_a) relative to the position of the center of the receiving surface will have coordinates Q = [x_a, y_a, h(d)] with h(d) = 2d. The distance R can be calculated from

\[ R(x_s, y_s, x_a, y_a, d) = |PQ| \]

\[ = \sqrt{(2r_f + x_a - x_s)^2 + (y_a - y_s)^2 + (2d)^2}, \]

and the angles θ and ϕ are calculated from

\[ \theta(x_s, y_s, x_a, y_a, d) = \tan^{-1} \left[ \frac{y_a - y_s}{2r_f + x_a - x_s} \right] \]

and

\[ \phi(x_s, y_s, x_a, y_a, d) = \tan^{-1} \left[ \frac{2d}{\sqrt{(2r_f + x_a - x_s)^2 + (y_a - y_s)^2}} \right]. \]

The flux on the receiving fiber is, therefore,

\[ \Phi(d) = \int_{r_f}^{t_f} \int_{r_f}^{t_f} \int_{r_f}^{t_f} \int_{r_f}^{t_f} \frac{L(x_s, y_s, x_a, y_a, d) \cos^2 \theta}{R^2(x_s, y_s, x_a, y_a, d)} \times dx_s dy_s dx_a dy_a, \]

the shape of which can be seen in the example of Fig. 4 [trace (a)].
In the case of the single cut fiber displacement transducer in Fig. 2, the fiber end face is cut at an angle $\theta_{\text{cut}}$ with respect to its axis and the emitting light is redirected. The face of the cut fiber has an elliptical shape with minor axis $r_f$ and major axis $r_f^v = r_f / \cos(\theta_{\text{cut}})$. The ray inside the emitting fiber incident on the cut surface at an angle $\theta$ will emerge at an angle given by Snell’s refractive index condition $B(\theta) = \sin^{-1}(n \sin \theta)$, where $n$ is the refractive index of the fiber, and $n_{\text{air}}$ taken as 1. The flux on the receiving fiber is again calculated after integrating the radiance over the transmitting area

$$
\Phi = \int_{S} \int_{A} \frac{L' \cos \theta' \cos \psi}{R^2} ds da.
$$

where $L'$ is the radiance function at the surface of the transmitting fiber, $\theta'$ is the exit angle of the ray from the transmitting surface, and $\psi$ now is equal to the exit angle $\theta'$. The radiance $L'$ is derived from the function $L$ evaluated at the position of the source area element $ds$ and at a direction $(\theta, \phi)$ such that $B(\theta) = \theta'$ and $\theta'$ and $\phi$ are calculated from the relative positions of the transmitting and receiving surface elements $ds$ and $da$.

In Fig. 2 we first assume a point source $P$ on the emitting cut fiber surface with coordinates $P = (x_s, y_s, 0)$. A point $Q$ at position $(x_a, y_a)$ relative to the position of the center of the receiving surface of the cut fiber will have coordinates $Q = [2r_f^v + l(d) + x_a, y_a, h(d)]$ where $l(d) = 2d \cos \theta_{\text{comp}}, h(d) = 2d \sin \theta_{\text{comp}}$, and $\theta_{\text{comp}} = 90 - \theta_{\text{cut}}$, which is the complementary angle to the cut angle. The distance $R$ can be calculated from

$$
R(x_s, y_s, x_a, y_a, d) = |PQ| = \sqrt{(2r_f^v + l(d) + x_a - x_s)^2 + (y_a - y_s)^2 + (2d \sin \theta_{\text{comp}})^2}.
$$

(7)

The angles $\theta'$ and $\phi$ are then calculated from

$$
\theta'(x_s, y_s, x_a, y_a, d) = \cos^{-1} \left( \frac{2d \sin \theta_{\text{comp}}}{R(x_s, y_s, x_a, y_a, d)} \right)
$$

(8)

and

$$
\phi(x_s, y_s, x_a, y_a, d) = \tan^{-1} \left( \frac{y_a - y_s}{2r_f^v + l(d) + x_a - x_s} \right).
$$

(9)

In order to calculate $\theta$ we can use $B^{-1}(\theta') = \theta$ so that $\theta = \sin^{-1}(n^{-1} \sin \theta')$. The flux on the receiving fiber is, therefore,

$$
\Phi(d) = \int_{-r_f^v}^{r_f^v} \int_{-r_f^v}^{r_f^v} \int_{-r_f^v}^{r_f^v} \int_{-r_f^v}^{r_f^v} \int_{-r_f^v}^{r_f^v} \frac{L(x_s, y_s, x_a, y_a, \theta, \phi) \cos^2 \theta'}{R^2} ds dx dy dz dx_a dy_a,
$$

(10)

as shown in Fig. 4 [trace (b)].

A similar analysis to the one presented for single cut fibers can be performed for the double cut fibers. The double cut fibers are essentially repositioned single cut fibers so that their cut face is perpendicular to the reflector. The second cutting which is made along the fiber axis ensures improved light coupling for small distances between the fiber surface and the reflector as shown in Fig. 3. For $P = (x_s, y_s, 0)$, $Q = (x_a, y_a, h(d))$, and $h(d) = 2d$, the distance $R$ can be calculated from Eq. (2). The angles $\theta'$ and $\phi$ are then calculated from

$$
\theta'(x_s, y_s, x_a, y_a, d) = \cos^{-1} \left( \frac{2d}{R(x_s, y_s, x_a, y_a, d)} \right)
$$

(11)

and

$$
\phi(x_s, y_s, x_a, y_a) = \tan^{-1} \left( \frac{y_a - y_s}{x_a - x_s} \right).
$$

(12)
The analysis thereafter is similar to the one for the single cut fibers, the only difference being the boundaries for the integration

\[ \Phi(d) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{L(x, y, \theta, \phi) \cos^2 \theta'}{R(x, y, x_a, y_a, d)} \times dx \, dy \, dx_a \, dy_a, \]

as shown in Fig. 4 [trace (c)].

III. DISCUSSION

The radiometric analysis shows that uncut fiber sensors show the largest range (1200 µm for less than 1% linearity error) but a small responsivity (approximately 180 V m⁻¹ when a transimpedance amplifier with a 100 kΩ feedback resistor is used). Single cut fiber sensors exhibit an improvement in responsivity over the uncut sensor by a factor of 4 but this is achieved at the expense of range (650 µm). A further increase in responsivity by a factor of 64 over that of the uncut sensor is observed for the double cut configuration. This, however, reduces its operational range to only 100 µm. The double cut configuration is, therefore, particularly suitable in applications where feedback action is applied to the moving reflector surface.

If the coupler restricts the propagation of some of the modes, the distribution of light that exits the transmitter will be different to the one originally assumed. This would result in a shift of the maximum of the displacement versus coupling curve to larger distances and a reduction in the responsivity of the transducer. Misalignment between the probe and the reflector can be taken into account in the flux equations after assuming that \( \theta \neq \theta' \) and \( \phi \neq \omega \). The radiometric analysis may also be used in other applications such as refractometry or turbidity measurements, where these fiber configurations are used at a fixed reflector distance. Intensity referencing techniques suitable for these sensor configurations have been discussed elsewhere.